Factor Premium in Idiosyncratic Volatility

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Abstract

Residuals from linear factor-based asset pricing models exhibit volatility comovement in both the physical and the risk-neutral world. The volatility-factor structure in the individual stock variances cannot be replicated by assuming factors on the stock returns. Equalizing the market variance and the index variance shows that if a two-factor structure on individual variance exists, the common factor besides the market variance factor, the so-called *"common idiosyncratic variance factor"*, embeds a premium, which, combined with the correlation premium, explains the index variance risk. Empirical study using option prices and equity returns suggests a negative correlation between the premium of this factor and the market variance factor, which causes the insignificant variance premium on individual stocks. Further evidence shows that the trading strategy based on collecting this factor premium generates significantly positive returns.

JEL Classifications: G12, G13, C58.

Keywords: Comovement, factor loadings, variance beta, Kalman Filter.

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1 Introduction

Linear factor-based asset pricing models such as the CAPM of Merton (1973) and the APT of Ross (1976) suggest that the equity returns can be explained by the market returns or a linear combination of factors, leaving the residuals to be idiosyncratic, or firm specific. The factors represent the source of systematic risk, thus embed premia. The unexplained residuals from the linear equation are usually regarded as the idiosyncratic risk, which is not priced under perfect diversification. The theory also implies that the idiosyncratic volatility should not be priced. However, empirical tests about the asset pricing implication of the idiosyncratic volatility suggest otherwise. The well-known idiosyncratic volatility puzzle of Ang *et al* (2009) indicates that the high residual variances from a Fama-French (1993) model imply low future returns, which contradicts to the theory that the idiosyncratic risk should be irrelevant or at least positively related to returns under imperfect diversification. This puzzle calls for attention on the pricing implication of the idiosyncratic volatility.

Although many studies have been focusing on explaining this puzzle such as Chen and Petkova (2012), Fu (2009) and Cao and Han (2013), where missing factors are regarded as the dominating reason for the priced idiosyncratic risk, it still draws attention as to why, even though the co-variations in the returns are almost totally extracted by the return factors, the idiosyncratic variances of different stocks still possess a high level of comovement, see for example Herskovic *et al*. (2014). The comovement in idiosyncratic volatility can only be partially explained by missing factors, as they show that even after a 5-factor PCA, where 98% of the variation in raw return series has been explained, the residual variances of different stocks still exhibit strong correlations. This finding calls for analysis on four topics: 1) why, statistically or economically, does the comovement exist; 2) as it cannot be explained by the linear factor pricing models, how to capture this feature; 3) if the comovement in idiosyncratic volatility can be modelled by a factor, will this factor be priced; 4) if the factor is priced, how to trade the factor premium. This paper aims at explaining these four topics.

To understand the comovement in idiosyncratic volatility, we start our analysis using the daily return series of 30 constituents in the Dow Jones Industrial Average Index. We find similar results as Herskovic *et al*. (2014) that the variances of the residuals from the static and conditional CAPM co-move at very high levels of correlations. After extending this analysis to the risk-neutral world where we use the standardized at-the-money implied volatilities of the 30 stocks, we find even higher levels of comovement. We proceed by using the variance beta instead of the squared return beta to extract the market variance. The result shows that this method decreases the comovement significantly such that the variance residual series only a weak comovement in the absolute terms. This finding sheds light on our second topic, in which we are tempted by using the volatility-factor model to explain this feature. The comovement in absolute terms simply suggests different signs of factor loadings on the possible common idiosyncratic variance factors, which combined with the market variance factor, forms a two-factor model on the volatility structure. We claim that there are three reasons for the comovement in idiosyncratic variances: 1) the difference between the variance beta and the squared return beta, 2) the existence of a volatility factor on which some of the stocks have negative factor loadings, and 3) a zero-correlation pricing factor that contributes no correlation but similar volatility to different stocks.

The second part of the paper aims at applying the volatility-factor model implied by the first two reasons. We apply the volatility-factor GAS model of Boswijk and Liu (2014) on the equity returns, and find sufficient evidence of the existence of a second volatility factor besides the market variance factor, which we call the common idiosyncratic variance factor, or the CIV factor following Herskovic *et al*. (2014). Across different sample groups, about 1/3 of all the stocks have negative loadings on the CIV factor.

To investigate the premium of this volatility factor, we apply Kalman Filter on the implied volatility series and extract the conditional and unconditional factor premia for the market variance factor and the CIV factor. In all of our sample groups, the market variance factor contains a positive premium, which is in line with empirical findings. The CIV factor, however, contains a negative premium in most groups. Considering the fact that most stocks have insignificant variance premium, the negative premium is economically intuitive as it serves as offsetting the market variance premium.

As the next step, we follow the method in Driessen *et al*. (2009) and design a trading strategy that involves long positions in individual straddles and the S&P500 Index, and short positions in individual stocks and the index straddle. The strategy, under appropriate portfolio weight, has only positive exposures to the CIV factor, thus collecting the factor premium. Throughout the sample period, the trading strategies across different sample groups achieve daily excess returns of around 1.5% and monthly returns above 20%, which is a strong evidence of the negative CIV factor premium.

This paper develops as follows: Section 2 describes the data used in the physical and the risk-neutral world; Section 3 provides the intuition as to why we need volatility-factor models to analyse the comovement in idiosyncratic variances; Section 4 derives the impact of the CIV factor premium on individual variances and index variances; Section 5 implements the factor filtration and premia estimation; Section 6 shows the details and empirical performance of the trading strategy that only trades the CIV factor; Section 7 concludes the paper.

2 Data Description

The daily stock price series are obtained from the CRSP database with all current 30 constituents in the Dow Jones Industrial Average Index. The data window ranges from 3rd January 2000 to 31st December 2013. The market index is taken to be the S&P500 Index. The implied volatility series and the option price series are obtained from the OptionMetrics database, where standardized at-the-money (ATM) implied volatility series with 30-day maturity is taken for each individual stock and the S&P500 Index from 3rd January 2000 to 31st August 2013. We follow the data cleaning procedure of Driessen *et al*. (2009) and only focus on short maturities from 14 to 60 days: first, we delete the option prices with missing implied volatilities and zero open interests; second, we delete the options with extreme moneyness such that the Black-Scholes delta is below 0.15 for calls and above -0.05 for puts; third, we only keep the options that can be formed to a straddle contract.

3 Comovement in idiosyncratic volatility

Residuals from linear factor-based asset pricing model exhibit statistical independence, but a strong factor structure in the squared terms, i.e., a strong comovement feature in their idiosyncratic volatilities. This section provides empirical evidences toward the idiosyncratic volatility comovement in both the physical ($\mathbb{P}$) and the risk-neutral ($\mathbb{Q}$) world. A more detailed $\mathbb{P}$-world study can be found in Herskovic *et al*. (2014). We focus on the CAPM so that the same analysis can be conducted in the risk-neutral world using the implied beta calculation of Buss and Vilkov (2012).

3.1 Comovement in $\mathbb{P}$ world

To examine the comovement in the idiosyncratic volatilities under the CAPM setting in the $\mathbb{P}$ world, we use the daily return series of the 30 stocks together with the daily return series on the S&P500 Index. We calculate the firm specific betas using the static sample averages and the multivariate DCC method of Engle (2002), which respectively corresponds to the static and the conditional CAPM. Before extracting the market return, the average pairwise correlation of the daily stock return series is 0.3957. In the static case, we calculate the constant beta for stock i by $\hat{\beta}_i = \sum_{t=1}^{T} r_{it} r_{mt} / \sum_{t=1}^{T} r_{mt}^2$. In the dynamic case, we apply the bivariate DCC model on each stock return series and the market return series. The conditional beta for stock i is calculated by $\hat{\beta}_{it} = \rho_{it}\sigma_{i,t}/\sigma_{mt}$, where ρ_{it} is the conditional correlation between the daily return of stock i and the market return. Here σ^2_{mt} is the conditional variance of the market return. The same method is also used by Hansen and Lunde (2014). The return residuals in both cases are then calculated by

$$\text{static}: \hat{\epsilon}_{it} = r_{it} - \hat{\beta}_i r_{mt} \quad \text{dynamic}: \hat{\epsilon}_{it} = r_{it} - \hat{\beta}_{it} r_{mt}$$

The 30 residual series $\{\epsilon_{it}\}_{t=1}^{T}, \quad i = 1, ..., 30$ exhibit an average pairwise correlation of 0.0089 in the static beta case and 0.0065 in the dynamic beta case, i.e., in both cases, the market return extracts over 97% of the return correlation.

In the static case, we apply the GJR-GARCH(1,1) model on each series $\{\epsilon_{it}\}_{t=1}^{T}$, thus collecting the conditional variances of the residuals, or the idiosyncratic variances for each stock i. The pairwise average correlation of the idiosyncratic variance series of 30 stocks is 0.5296, which is much higher than the residual correlation 0.0089. In the dynamic case, we calculate the idiosyncratic variance of stock i by $\sigma^2_{it} - \hat{\beta}^2_{it}\sigma^2_{mt}$, and the series exhibits an average pairwise correlation of 0.5208. Strong comovement in idiosyncratic variances are found in both the static and the conditional CAPM settings, which suggests the potential factor structure on the idiosyncratic variances, or a two-factor structure on the individual variances with the first factor being the market return variance. This result is not innovative since one can find a more comprehensive analysis on linear pricing models in the work of Herskovic *et al.* (2014). Therefore, we did not go into more complicated linear pricing models to show the comovement and give all the credit of this subsection to Herskovic *et al.* (2014). However, the following subsections are considered our contribution in ameliorating this finding.

3.2 Comovement in $\mathbb{Q}$ world

In this subsection, we extend Herskovic *et al*'s work to the risk-neutral world, and see if the comovement in idiosyncratic volatility still preserves. The existence of comovement in implied idiosyncratic volatilities will suggest that the volatility-factor structure preserves through the physical and the risk-neutral world, which makes it tempting to investigate whether this volatility-factor structure conveys a variance risk premium. To examine the comovement in the implied idiosyncratic volatility, we use the standardized implied volatilities of the at-the-money options with 30-day maturity for the 30 stocks and the S&P500 Index. To calculate the idiosyncratic variances, we first calculate the implied betas using the method of Buss and Vilkov (2012), where

$$\beta^Q_{iM,t} = \frac{\sigma^Q_{i,t}\sum_{j=1}^{N}\omega_j\sigma^Q_{j,t}\rho^Q_{ij,t}}{(\sigma^Q_{M,t})^2} \tag{3.2.1}$$

where $\sigma^Q_{i,t}$ is the implied volatility of stock i at time t and $\sigma^Q_{M,t}$ is the implied volatility of the S&P500 Index at time t; ω_j is the weight given to stock j in the index, which for simplicity, we restrict it to be 1/30; $\rho^Q_{ij,t}$ is the implied conditional correlation between stock i and stock j at time t. Note that $\rho^Q_{ij,t}$ is the only variable that cannot be collected in the Q-world, therefore, Buss and Vilkov (2012) propose calculating $\rho^Q_{ij,t}$ by

$$\rho^Q_{ij,t} = \rho^P_{ij,t} - e_t(1 - \rho^P_{ij,t})$$

where $\rho^P_{ij,t}$ is the conditional correlation under P and is calculated by a 100-day rolling window in the previous subsection. A negative e_t implies a positive correlation premium ($\rho^Q_{ij,t} > \rho^P_{ij,t}$). α_t can be calculated by using the condition

$$(\sigma^Q_{M,t})^2 = \sum_{i=1}^{N} \omega_i^2 (\sigma^Q_{i,t})^2 + \sum_{i=1}^{N}\sum_{j=1}^{N} \omega_i \omega_j \sigma^Q_{i,t} \sigma^Q_{j,t} \rho^Q_{ij,t}$$

then

$$e_t = -\frac{(\sigma^Q_{M,t})^2 - \sum_{i=1}^{N}\sum_{j=1}^{N} \omega_i \omega_j \sigma^Q_{i,t} \sigma^Q_{j,t} \rho^P_{ij,t}}{\sum_{i=1}^{N}\sum_{j=1}^{N} \omega_i \omega_j \sigma^Q_{i,t} \sigma^Q_{j,t} (1 - \rho^P_{ij,t})}$$

The implied idiosyncratic variance $(\sigma^{Qi}_{it})^2$ is then calculated as

$$(\sigma^{Qi}_{it})^2 = (\sigma^Q_{i,t})^2 - (\sigma^Q_{M,t})^2 (\beta^Q_{iM,t})^2$$

Performing this procedure on each of the 30 stocks, we can collect the firm specific idiosyncratic variances implied from the $\mathbb{Q}$-world CAPM. Again, we calculate the average pairwise correlation between the idiosyncratic variance series, and the result is 0.6710, which implies that the comovement in the risk-neutral world is even higher than in the physical world. As a clear illustration, Figure 1 shows the idiosyncratic variances from 10 selected stocks in both the physical and the risk-neutral world.

[Insert Figure 1 about here]

3.3 The variance beta

From the conditional CAPM equation, one can derive a similar form for the conditional variance,

$$\sigma^2_{it} = \beta^2_{it} \sigma^2_{mt} + \sigma^2_{it,I} \tag{3.3.1}$$

such that the individual variances are driven by the market variance and the idiosyncratic variance. We can consider σ^2_{mt} as the market variance factor. The factor loading derived from the CAPM is then β^2_t. Note that the factor loading is calculated under the restrictions that the residual or the idiosyncratic variance $\sigma^2_{it,I}$ from this equation should always be positive. Relaxing the positive-residual restriction will enable the market variance factor to extract more common features in individual variances, thus reducing the comovement in the residuals. Note that when the residuals are not positive, we cannot use the term idiosyncratic variance or idiosyncratic volatility, therefore we use the term variance residual instead.

One way to relax the restriction is using the variance beta introduced by Carr and Wu (2009). In the static case, we calculate the firm specific beta $\hat{\beta}_{iv}$ by running the regression

$$\sigma^2_{it} = \alpha_i + \beta_{iv}\sigma^2_{mt} + v_{it}$$

The variance residual series $\{v_{it}\}_{t=1}^{T}$, $i = 1, ..., 30$ are collected. In the dynamic case, we use a rolling window regression method and calculate the conditional variance beta by $\hat{\beta}_{iv,t}$ for firm i at time t by running the regression

$$\sigma^2_{is} = \alpha_{it} + \beta_{iv,t}\sigma^2_{ms} + v_{is}, \quad s = t-99, ..., t$$

The variance residual will be likely to contain negative entries when $\beta_{iv} > \beta_i^2$, in which case more comovement in the individual variances are explained by the market variance than in the linear pricing model. Performing this procedure on both the conditional variances and the implied variances, the variance residual average correlations are 0.2704 (static) and 0.1884 (dynamic)in the $\mathbb{P}$ world and 0.3515 (static) and 0.3195 (dynamic) in the $\mathbb{Q}$ world. Figure 2 shows the variance residuals under the dynamic variance beta. Compared with the idiosyncratic variances in the CAPM case, we can see that the comovement feature in the variance residuals is much weaker.

[Insert Figure 2 about here]

3.4 Reason of comovement

We start the analysis using the optimal orthogonal portfolio introduced by MacKinlay (1995), in which individual stock returns can be modelled as

$$r_t = Bf_{pt} + \beta_h f_{ht} + u_t$$

where f_{pt} is the K by 1 vector of time-t factor portfolio excess return. In cases where the K-factor portfolios cannot be combined to form the tangency portfolio, we should include portfolio f_h, the optimal orthogonal portfolio, so that the constant disappears. The combination of f_p and f_h then forms the tangency portfolio. This equation can serve as the general case of linear factor-based pricing models, and the factors f_h can be the possible omitted factor. Taking the conditional variance of r_t, we have

$$\sigma_t^2 = \text{diag}(B\text{var}(f_{pt})B') + \beta_h^2\sigma_{ht}^2 + \sigma_{ut}^2 \tag{3.4.1}$$

where $\text{var}(f_{pt})$ and σ_{ht}^2 can be treated as the common volatility factors.

3.4.1 The difference between the betas

Equation (3.4.1) and (3.3.1) both show that the linear factor-based pricing model restricts the variance residual to be positive, which causes small factor loadings on the variance factors, such as β_h^2 in equation (3.4.1). The smaller correlation and weaker comovement in the variance residuals under the variance beta cases are the results of the difference in variance betas and the squared return betas. Figure 3 shows the scatter plot of the variance betas versus the squared return beta for all the 30 stocks. In 28 out of 30 stocks, the variance betas are higher, meaning that in most cases the squared return betas fail to fully extract the comovement caused by the market variance in the variances. Therefore, one reason of the comovement in the variance residual from a linear pricing model is that the squared return betas cannot serve as the variance betas, since the former should always maintain positive residuals.

[Insert Figure 3 about here]

The fact that variance betas are able to extract more comovement of the variance residual series than the squared return betas calls for the class of volatility-factor model which assumes factor structures on the volatility or variance directly. Under the CAPM framework, we can model the individual variance using a single volatility-factor model, where the only factor that drives all the individual variances is the market variance. The factor loadings on the market factor are the variance betas, and the residuals are allowed to be negative.

3.4.2 Negative loading on volatility factor

From equation (3.4.1), one feature of the missing factor f_{ht} is that the loading β_h will be squared in the variance equation, therefore only positive loading can be observed in the corresponding volatility factor σ_{ht}^2, which also implies the equivalence of volatility-factor models and the return-factor models. However, if the loadings on the volatility factors have different signs across different stocks, one can no longer derive the return factor structure as the loading of the return factor will be a complex number. Intuitively, the factor pricing model can only detect the volatility factor(s) that has the same impact (in sign) on all individual stocks, but fails to capture the factor having different impacts (in sign).

From Figure 2, one can see that even though the variance residuals do not show a clear comovement, they tend to move together in magnitude, i.e., the absolute values of the residuals tend to move together. In fact, as illustrated in Table 1, the correlations of the absolute values of the variance residuals are, in $\mathbb{P}$ world, 0.3552 for the static case and 0.3760 for the dynamic case, and in $\mathbb{Q}$ world, 0.4110 for the static case and 0.4304 for the dynamic case, all four correlations are larger than the their correspondences in the non-absolute value cases. This is a clear indication of the existence of another volatility factor, on which different stocks might have different factor loadings in signs.

[Insert Table 1 about here]

3.4.3 Zero-correlation pricing factor

Another possible explanation is the existence of a zero-correlation pricing factor. Statistically, assuming an N-vector return r_t follows

$$r_t = \beta r_{mt} + W_t r_{xt} + \gamma \epsilon_t, \;\; r_{mt} \sim N(0,1), \;\; r_{xt} \sim N(0,1), \;\; \epsilon_t \sim N(0,I)$$

where W_t is a N by 1 vector with element values being either 1 or -1. Every element of W_t is randomly chosen with equal probabilities assigned to both values. Since the only correlation between r_t should come from r_{mt}, we apply a 1-factor PCA on the return vector. We take $N = 30$, $\beta = 1 : 0.02 : 5$ and $\gamma = 0.02 : 0.02 : 2$, so that β is large enough for us to use one principal component as the factor. For each pair of β and γ, we calculate the correlation of the returns, the correlation of the squared returns, the correlation of the residuals and the correlation of the squared residuals.

[Insert Figure 4 about here]

Figure 4 shows the graph of the four different correlations given different combinations of β and γ. When γ is small, i.e., the noise in return observations is small, all but the correlation of the PCA residuals have high correlations. The squared residuals, which we are particularly interested in, display very high correlations, which are not influenced much by the level of β. Combined with the bottom left graph, one can see that even though the PCA has extracted nearly all the correlation in the returns, the comovement in the squared residuals cannot be extracted, since the linear pricing factor simply cannot detect such a factor as r_{xt} with factor loadings W_t. This forms another situation as to why there exists comovement in the variance residuals.

4 Factor premium on idiosyncratic variance

The analysis above shows that some of the comovement in the variance residuals can be explained by another volatility factor on which each stock can have positive or negative factor loadings. We follow this setting and see if this factor embeds a premium. To begin with, we assume that the instantaneous variance of individual stock i follows

$$\phi_{it}^2 = \delta_{Ii}\phi_{mt}^2 + \delta_{xi} f_t + \alpha_i \tag{4.0.2}$$

where ϕ_{mt}^2 is the instantaneous market variance or the index variance, which can be treated as the market variance factor with factor loading δ_{mi}. f_t is the second volatility factor with factor loading δ_{xi}, here we call f_t the common idiosyncratic variance factor, or the CIV factor following Herskovic *et al.* (2014). α_i is a firm specific constant that captures the unconditional mean for the variance residuals. With no loss of generality, we assume $E^{\mathbb{P}}[f_t] = 0$ and $cov^{\mathbb{P}}(\phi_{It}^2, f_t) = 0$, thus the local level of the variance residuals is totally determined by α_i.

From the equation of index variance in Driessen *et al.* (2009)

$$\phi_{It}^2 = \sum_{i=1}^{N} \omega_i^2 \phi_{it}^2 + \sum_{i=1}^{N} \sum_{j \neq i} \omega_i \omega_j \phi_{it} \phi_{jt} \rho_{ij,t} \tag{4.0.3}$$

we know that the index variance changes are driven by shocks to both individual variances $\phi_t(t)^2$ and the correlation $\rho_{ij,t}$. If we treat the index variance as the market variance, i.e., $\phi_{It}^2 = \phi_{mt}^2$, then the individual variance changes are also driven by the index variance. Moreover, we assume a factor structure on $\rho_{ij,t}$, such that

$$\rho_{ij,t} = \theta_{ij}\bar{\rho}_t \tag{4.0.4}$$

A more restricted version will be applying the equicorrelation model of Engles and Kelly (2009), where $\rho_{ij,t} = \bar{\rho}_t$.

Assuming constant index weights $\{\omega_i\}$ for stock i and defining $\iota_i = \omega_i^2 + \sum_{j \neq i} \omega_i \omega_j \frac{\phi_j}{\phi_i} \rho_{ij}$, we substitute equation (4.0.2) and (4.0.4) in equation (4.0.3) for $\phi_i(t)^2$ and $\rho_{ij}(t)$, and apply Ito's lemma on the variance risk premium $E_t^{\mathbb{Q}}[d\phi_I^2] - E_t^{\mathbb{P}}[d\phi_I^2]$:

$$E_t^{\mathbb{Q}}[d\phi_I^2] - E_t^{\mathbb{P}}[d\phi_I^2] = \gamma_f \left\{ E_t^{\mathbb{Q}}[df] - E_t^{\mathbb{P}}[df] \right\} + \gamma_\rho \left\{ E_t^{\mathbb{Q}}[d\bar{\rho}] - E_t^{\mathbb{P}}[d\bar{\rho}] \right\} \tag{4.0.5}$$

where $\gamma_f = \sum_{i=1}^{N} \frac{\iota_i \delta_{xi}}{1 - \sum_{i=1}^{N} \iota_i \delta_{mi}}$, and $\gamma_\rho = \sum_{i=1}^{N} \sum_{j \neq i} \frac{\omega_i \omega_j \phi_i(t) \phi_j(t)}{1 - \sum_{i=1}^{N} \iota_i \delta_{mi}}$ are the parameters that measure the speed of mean reversions. Equation (4.0.5) shows that the index variance risk premium is consisted of the CIV factor premium and the correlation premium.

From equation (4.0.2), we have

$$E_t^{\mathbb{Q}}[d\phi_i^2] - E_t^{\mathbb{P}}[d\phi_i^2] = \delta_{Ii}\left\{E_t^{\mathbb{Q}}[d\phi_I^2] - E_t^{\mathbb{P}}[d\phi_I^2]\right\} + \delta_{xi}\left\{E_t^{\mathbb{Q}}[df] - E_t^{\mathbb{P}}[df]\right\} \tag{4.0.6}$$

such that the individual stock variance risk premium is determined by the sum of index variance premium and the CIV factor premium. Empirically, positive volatility risk premium is found in the index, while in individual stocks, the premia are not significantly positive, sometimes even negative. From Equation (4.0.6) we can see that if δ_{xi} is positive, or positive for most stocks, the relation (4.0.6) implies a negative premium on the CIV factor $f(t)$ that offsets the market variance premium.

5 Factor filtration and premium estimation

We have shown that the CIV factor, if exists, should have a negative factor premium given positive factor loadings. In this section, we propose ways to filter the CIV factor in both the physical and the risk-neutral world, so that the difference between them can be detected together with the evidence of factor premium.

5.1 Physical world filtering

Assuming there are N risky assets in the market and the index return is denoted by r_{mt}. To filter the CIV factor, we model the $N+1$ observed daily return series using the volatility-GAS model of Boswijk and Liu (2014). The vector of conditional variances is denoted by Δ_t, and

$$\Delta_t = \begin{pmatrix} \sigma_{1t}^2 \\ \sigma_{2t}^2 \\ \vdots \\ \sigma_{Nt}^2 \\ \sigma_{mt}^2 \end{pmatrix} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_N \\ \alpha_m \end{pmatrix} + \begin{pmatrix} \delta_{m1} & \delta_{x1} \\ \delta_{m2} & \delta_{x2} \\ \vdots & \vdots \\ \delta_{mN} & \delta_{xN} \\ \delta_m & 0 \end{pmatrix} \begin{pmatrix} f_{mt} \\ f_t \end{pmatrix}$$

Both factors are assumed to have unit variance and zero mean, s.t.,

$$\begin{pmatrix} f_{mt} \\ f_t \end{pmatrix} = (I - B^2)^{1/2} \begin{pmatrix} s_{m,t-1} \\ s_{t-1} \end{pmatrix} + B \begin{pmatrix} f_{m,t-1} \\ f_{t-1} \end{pmatrix}$$

This setting implies that f_{mt} is the market variance factor and f_t is the CIV factor. Note that the market variance factor f_{mt} originated as the market return factor, while the second factor f_t serves as the factor that cannot be detected from the return-factor structure as it is possible to have a negative loading δ_{xi}. We would expect that f_{mt} to be proportional to the market variance σ_{mt}^2 while f_t is somewhat different from f_{mt}.

When the number of asset increases, the correlation parameters increases by the rate of $(N-1)/2$. Therefore, for simplicity, we assume a single factor driving all the cross sectional correlation, which resembles the equicorrelation model of Engle and Kelly (2009). In the factor model, it is formulated as

$$\text{vech}_L(R_t) = \bar{\rho}_t$$

where R_t is the conditional correlation matrix and $\bar{\rho}_t$ serves as the market-wide correlation factor. The dynamics of $\bar{\rho}_t$ can be more flexible, for example:

$$\bar{\rho}_t = \omega_\rho(1-b_3) + a_\rho s_{\rho,t-1} + b_\rho \bar{\rho}_{t-1}$$

where $s_{\rho,t-1}$ is the scaled score function as introduced in Creal *et al.* (2013). The total number of parameters is then $3N+7$. The parameters can be estimated by the method of maximum likelihood assuming

$$r_t|\mathcal{F}_{t-1} \sim N(0,\Sigma_t) \text{ and } \Sigma_t = D_t R_t D_t$$

where $D_t = \text{diag}(\Delta_t^{1/2})$. The detailed calculation about the score function and the scaling matrix can be found in Boswijk and Liu (2014), where the volatility-factor GAS model is introduced based on the multivariate GAS model of Creal *et al.* (2011).

We start the $\mathbb{P}$ world estimation using the 30 constituents in the Dow Jones Industrial Average Index. As for the market return, we choose the S&P500 Index return. All observations are collected on daily base. We first divide the 30 stocks in three groups of 10 stocks, so that the volatility-factor GAS model is able to converge in a reasonable time. The sample window ranges from 3rd Jan 2000 to 31st Dec 2013. We perform the estimation process under three sorting rules. In the first rule, we divide the 30 stocks into three different groups sorted by their market capital sizes, resulting the small, medium and large groups. The second rule sorts the stocks by their unconditional variances estimated by univariate GJR-GARCH(1,1) model, resulting the low, medium and high variance groups. In the third rule, we randomly divide the 30 stocks into three groups, just for robust study. In Table 2, Table 3, and Table 4 we present the estimation results for the three sorting methods. We can see that in all three cases, significantly negative loadings on f_t can be found for some stocks, whereas the loadings on the market variance factor are always positive. In Figure 5, we present the estimated factors for all three sorting rules and all three groups. We can see that in most part of the data window, the CIV factors across different groups are very much alike and exhibit opposite and more persistent paths compared with the market variance factors after the crisis.

[Insert Table 2 about here]

[Insert Table 3 about here]

[Insert Table 4 about here]

5.2 Risk-neutral world filtering

In this section, we conduct the estimation of the CIV factor f_t under the risk-neutral world. Define by equation (5.2.1) and (5.2.2) the process of the implied variance

$$(\sigma_{it}^{\mathbb{Q}})^2 = \alpha_i^{\mathbb{Q}} + \delta_{mi}^{\mathbb{Q}} f_{mt}^{\mathbb{Q}} + \delta_{xi}^{\mathbb{Q}} f_t^{\mathbb{Q}} + v_{it}^{\mathbb{Q}} \tag{5.2.1}$$

$$(\sigma_{mt}^{\mathbb{Q}})^2 = \alpha_m^{\mathbb{Q}} + \delta_m^{\mathbb{Q}} f_{mt}^{\mathbb{Q}} + v_{mt}^{\mathbb{Q}} \tag{5.2.2}$$

where $v_{it}^{\mathbb{Q}}$ and $v_{mt}^{\mathbb{Q}}$ capture the unexplained residuals from the two-factor structure and are assumed to be zero-mean and have no serial dependence. Follow Christoffersen *et al.* (2013), Duan and Wei (2009) and Serban *et al.* (2008), we assume that the factor loadings preserves through measure change, therefore $\delta_{mi}^{\mathbb{Q}} = \delta_{mi}$, $\delta_{xi}^{\mathbb{Q}} = \delta_{xi}$ and $\delta_m^{\mathbb{Q}} = \delta_m$. We also restrict $\alpha^{\mathbb{Q}} = \alpha^{\mathbb{P}}$, so that all the variance premia come from the two factors. For stock i, the unconditional variance risk premium λ_i is then

$$\lambda_i = E^{\mathbb{Q}}[(\sigma_{it}^{\mathbb{Q}})^2] - E^{\mathbb{P}}[(\sigma_{it}^{\mathbb{P}})^2] = \delta_{mi}\mu_m^{\mathbb{Q}} + \delta_{xi}\mu_f^{\mathbb{Q}} \tag{5.2.3}$$

where $\mu_m^{\mathbb{Q}} = E^{\mathbb{Q}}[f_{mt}^{\mathbb{Q}}]$, $\mu_f^{\mathbb{Q}} = E^{\mathbb{Q}}[f_t^{\mathbb{Q}}]$, $\mu_m^{\mathbb{P}} = E^{\mathbb{P}}[f_{mt}^{\mathbb{P}}] = 0$, and $\mu_f^{\mathbb{P}} = E^{\mathbb{P}}[f_t^{\mathbb{P}}] = 0$. Conditionally, set the variance risk premium at time t to be λ_{it}, then

$$\lambda_{it} = \delta_{mi} \underbrace{\left(E^{\mathbb{Q}}\left[f_{mt}^{\mathbb{Q}} | \mathcal{F}_{t-1}\right] - RF_{m,t}\right)}_{\lambda_{f_m,t}} + \delta_{xi} \underbrace{\left(E^{\mathbb{Q}}\left[f_t^{\mathbb{Q}} | \mathcal{F}_{t-1}\right] - RF_{f,t}\right)}_{\lambda_{i,xt}} \tag{5.2.4}$$

where $RF_{m,t}$ and $RF_{f,t}$ are the realized values of $f_{mt}^{\mathbb{P}}$ and $f_t^{\mathbb{P}}$ at time t. From equation (5.2.4), we can see that the variance risk premium of stock i is consisted by first the market variance factor premium and second the CIV factor. From the P-world estimation we know that the estimated δ_{mi} is always positive for all stocks, which combine with the significant positive premium in index variances and insignificant premium in individual stock variances, implies that $\delta_{xi}\lambda_{i,xt}$ should be negative for most stocks. Since over 2/3 of the 30 stocks show a positive δ_{xi}, we conjecture that $\lambda_{i,xt}$ should be negative, that is, the CIV factor carries a negative premium.

To test this premium, we use the implied volatility data from OptionMetrics. The daily time series of the implied volatilities for each standardized individual stocks with 30 days maturity are collected together with the implied volatility of the S&P500 Index. Therefore, we have 31 daily time series of implied volatilities, each of which measures the expected volatility for the next 30 days (21 trading days on average). We take the P-world filtered factor values as the realized values, namely, the realized value for $f_t^{\mathbb{Q}}$ is then $\sum_{s=0}^{20} \hat{f}_{t+s}^{\mathbb{P}}$. Since we are taking the 1-month added value, the lagged effect in filtering $\hat{f}_t^{\mathbb{P}}$ can be ignored. One way to model the dynamics of $f_t^{\mathbb{Q}}$ and $f_{mt}^{\mathbb{Q}}$ is to model them separately using the sum of their corresponding realized values and a dynamic risk premium process. Define θ_{ft} and θ_{mt} as the two premium process for f_t and f_{mt}, then our conjecture suggests a positive unconditional mean for θ_{mt} and a negative one for θ_{ft}. Note that from equation (5.2.3) and (5.2.1), the two unconditional means should be equal to $\mu_m^{\mathbb{Q}}$ and $\mu_f^{\mathbb{Q}}$, respectively. Combined with equation (5.2.1) and (5.2.2), we construct a state-space model:

$$(\sigma_{it}^{\mathbb{Q}})^2 = \alpha_i^{\mathbb{P}} + \delta_{mi} f_{mt}^{\mathbb{Q}} + \delta_{xi} f_t^{\mathbb{Q}} + v_{it}^{\mathbb{Q}} \quad v_{i,t}^{\mathbb{Q}} \sim N(0, h_i) \tag{5.2.5}$$

$$(\sigma_{mt}^{\mathbb{Q}})^2 = \alpha_m^{\mathbb{P}} + \delta_m f_{mt}^{\mathbb{Q}} + v_{mt}^{\mathbb{Q}} \quad v_{m,t}^{\mathbb{Q}} \sim N(0, h_m) \tag{5.2.6}$$

$$f_{t+1}^{\mathbb{Q}} = \sum_{s=1}^{21} \hat{f}_{t+s}^{\mathbb{P}} + \theta_{f,t+1} \tag{5.2.7}$$

$$f_{m,t+1}^{\mathbb{Q}} = \sum_{s=1}^{21} \hat{f}_{m,t+s}^{\mathbb{P}} + \theta_{m,t+1} \tag{5.2.8}$$

$$\theta_{f,t+1} = \mu_f^{\mathbb{Q}}(1 - \psi_f) + \psi_f \theta_{f,t} + v_{f,t+1}^{\mathbb{Q}}, \quad v_{f,t+1}^{\mathbb{Q}} \sim N(0, h_f) \tag{5.2.9}$$

$$\theta_{m,t+1} = \mu_m^{\mathbb{Q}}(1 - \psi_m) + \psi_m \theta_{m,t} + v_{m,t+1}^{\mathbb{Q}}, \quad v_{m,t+1}^{\mathbb{Q}} \sim N(0, h_{fm}) \tag{5.2.10}$$

The model can be easily estimated by the Kalman Filter. One would argue that the normality assumption for $v_{i,t}^{\mathbb{Q}}$ is unrealistic and can lead to biased estimations of μ_m and μ_f. We would investigate this feature in a later study where the importance sampling method is applied to see the difference.

The parameters to be estimated are $(\{h_i\}_{i=1}^N, h_m, \mu_f^{\mathbb{Q}}, \mu_m^{\mathbb{Q}}, \psi_f, \psi_m, h_f, h_{fm})$, a total of $(N+7)$ parameters. We follow the previous section and perform the estimation for each group of stocks. This is reasonable given that the market loading δ_m's are different across groups. Table 5 shows the estimation results from the nine groups of stocks. One can see that the estimated unconditional premia for the market variance factor are always positive for the nine groups of stocks, with the highest premium found in the group of stocks with large market capital. The factor premia for the CIV factor are negative for all the three groups sorted by market capital size, and negative in 7 out of 9 groups, which corresponds to our conjecture that the CIV factor embeds a negative premium to offset the market variance factor premium. The persistent parameters of the market factor are smaller than the ones of the CIV factor, which is the same in the P-world estimation. In Figure 6, we show the filtered time-varying premia $\hat{\theta}_{mt}$ and $\hat{\theta}_{ft}$.

[Insert Table 5 about here]

[Insert Figure 6 about here]

5.3 The variance beta from factor loadings

In section 2, we show that one of the reasons that the variance residuals have comovement is that the squared return beta is smaller than the variance beta. In the two-factor volatility GAS model, the variance beta can be implied by the ratio δ_{mi}/δ_m. Table 6 shows the squared static return beta estimated using all the samples and the variance beta of each stock from all three sorting schemes. One can see that the estimated variance betas in the three schemes are very close to each other, and are consistently larger than the squared return betas. The differences between the variance betas and the squared return betas increase with the unconditional variances, i.e., the higher the unconditional variance, the more dependence on the market factor is omitted by the squared return beta.

[Insert Table 6 about here]

6 Trading strategy

In this section, we design the trading strategy that has only exposures to the CIV factor f_t. This strategy follows closely to the correlation trading strategy of Driessen *et al.* (2009) and involves positions on individual straddles, index straddles, individual stocks and the S&P500 Index.

To begin with, we assume that the unexpected shock on the individual stock price S_i follows

$$\frac{dS_i}{S_i} - E\left[\frac{dS_i}{S_i}\right] = \phi_i dB_i \tag{6.0.1}$$

and the index level follows

$$\frac{dS_I}{S_I} - E\left[\frac{dS_I}{S_I}\right] = \phi_I dB_I \tag{6.0.2}$$

where we assume B_i and B_I to be the standard Weiner processes. The symbol ϕ_i and ϕ_I are the same as in Section 4. Following the factor structure on the variances in the previous section, we can write

$$\phi_i^2(t) = \alpha_i + \delta_{mi} f_{mt} + \delta_{xi} f_t$$

$$\phi_m^2(t) = \alpha_m + \delta_m f_{mt}$$

Moreover, we have

$$d\phi_i^2 - E\left[d\phi_i^2\right] = \delta_{mi} df_{mt} + \delta_{xi} df_t$$

$$d\phi_I^2 - E\left[d\phi_I^2\right] = \delta_m df_m$$

Assuming O_i and O_I to be the price of the at-the-money straddles for each stock i and the index, then we have

$$\frac{dO_i}{O_i} - E\left[\frac{dO_i}{O_i}\right] = \frac{1}{O_i}\frac{\partial O_i}{\partial S_i}\phi_i S_i dB_i + \frac{1}{O_i}\frac{\partial O_i}{\partial \phi_i^2}\delta_{mi} df_m + \frac{1}{O_i}\frac{\partial O_i}{\partial \phi_i^2}\delta_{xi} df_x \tag{6.0.3}$$

$$\frac{dO_I}{O_I} - E\left[\frac{dO_I}{O_I}\right] = \frac{1}{O_I}\frac{\partial O_I}{\partial S_I}\phi_I S_I dB_I + \frac{1}{O_I}\frac{\partial O_I}{\partial \phi_I^2}\delta_m df_m \tag{6.0.4}$$

To derive the portfolio weights, we first restrict the weight on each individual straddle to be $1/N$, where N is the number of stocks in one group. In our empirical analysis, there are 10 stocks in each sorted groups, therefore, 10% of the initial wealth is invested in each individual straddle. We also apply a daily balancing strategy, so that the portfolio return is calculated each day by assuming the initial daily wealth to be 1. Besides the 100% investment in the individual straddles, the factor-trading strategy also contains weight z_i on individual stocks, z_I on the index, and y_I on the index straddle.

The delta-hedging conditions of each stock and the index require

$$\frac{1}{N}\frac{1}{O_i}\frac{\partial O_i}{\partial S_i}\phi_i S_i + z_i \phi_i = 0 \tag{6.0.5}$$

$$\frac{1}{O_I}\frac{\partial O_I}{\partial S_I}\phi_I S_I y_I + z_I \phi_I = 0 \tag{6.0.6}$$

Then the hedging of the market variance factor f_m provides us enough restriction to identify all the portfolio weights:

$$\frac{1}{N}\sum_{i=1}^{N}\frac{1}{O_i}\frac{\partial O_i}{\partial \phi_i^2}\delta_{mi} + \frac{1}{O_I}\frac{\partial O_I}{\partial \phi_I^2}\delta_m y_I = 0 \tag{6.0.7}$$

Denote the value of the portfolio as D, then

$$\frac{dD}{D} - E\left[\frac{dD}{D}\right] = \left(\frac{1}{N}\sum_{i=1}^{N}\frac{1}{O_i}\frac{\partial O_i}{\partial \phi_i^2}\delta_{xi}\right) df_x \tag{6.0.8}$$

This portfolio buys individual straddles and index, and shorts individual stocks and index straddle. The portfolio only has positive exposure to the CIV factor, thus collecting all the factor premium if the factor has a negative premium.

The trading strategy is implemented on a daily base with no transaction cost considered and zero interest rate assumed. We choose three groups of stocks sorted by market capital size and use the factor loadings δ_{mi}, δ_{xi} and δ_m in Table 2. On each day, we look for the calls and puts for each stock i with the same maturity and same strike, pick those with the maturity that generates the most number of options, and choose the one with moneyness closest to 1. The portfolio weight is calculated based on the delta and vega on the same day, and we keep the position until the next day. On the second day, we use the price of the straddles chosen from yesterday together with the price of stocks and index to calculate the daily portfolio return. We collect the return, reset the wealth to 1, and start choosing a new straddle. The reason we perform this daily balancing strategy is that the strategy is derived based on at-the-money straddle. Therefore, we keep the straddle position as close as possible to the current price. This strategy might cause a significant transaction cost, but we are more interested in an empirical evidence on the existence of the factor premium of f_t. We perform this strategy from January 3rd 2000 to August 31st 2013.

The results are illustrated in Table 7. One can see that the factor-trading strategy outperforms the other strategies by achieving significantly positive returns. The advantage is consistent throughout different groups. Note that the medium group achieves the highest return, which corresponds to the highest factor-premium $\hat{\mu}_f^{\mathbb{Q}}$ estimated in Table 5. Table 8 also presents the portfolio weights, where we can see approximately -35% of wealth is invested in the individual stocks, while around -160% is invested in the index straddle. The amount of money invested in the index is around 120%.

7 Conclusion

Residuals from the linear factor-based asset pricing models exhibit a strong feature of comovement. This comovement cannot be explained by omitting factors on the return structure. Assuming a volatility-factor based model, we conclude that the comovement in the variance residuals is due to the missing volatility factor and the difference between the variance beta and the squared return beta. We show that the missing volatility factor can be filtered in the $\mathbb{P}$ world using the volatility-factor GAS model. The filtered CIV factor exhibits a more persistent path than the market variance factor with robust results throughout different groups and sorting schemes. Second, we investigate the factor premium of the CIV factor using Kalman Filter using the implied volatility series. Preferable results are found where the conjecture of a negative CIV factor premium is supported. As a further evidence, we implement a daily trading strategy where the portfolio only has exposure to the CIV factor. The trading strategy outperforms some existing strategies by providing averaged daily returns around 1.5% and monthly returns higher than 20%, transaction cost unaccounted for. Some future work includes some positivity restrictions on the Kalman Filter approach and the implementation of a trading strategy considering the transaction cost.

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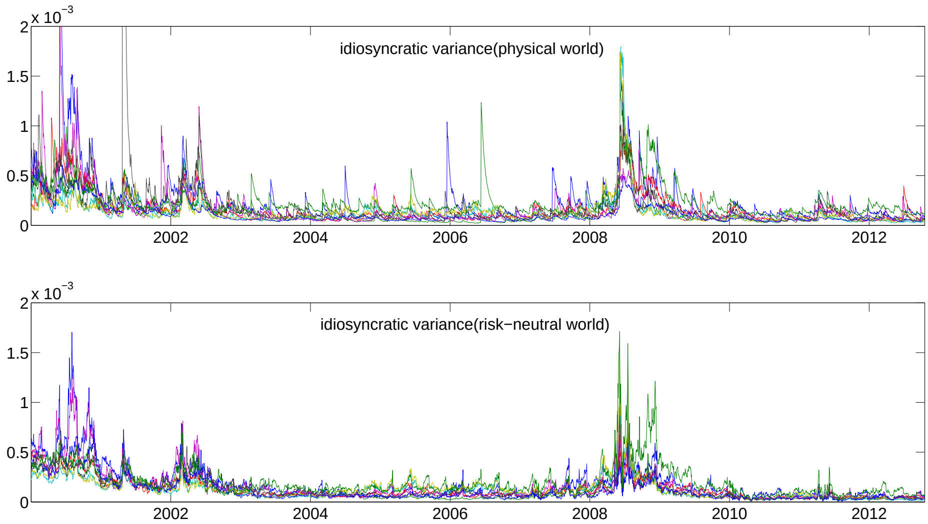

Figure 1: This figure shows the idiosyncratic variances for 10 selected stocks in P and $\mathbb{Q}$ world. The squared return betas are used in both cases. Data window ranges from 3rd January 2000 to 31 August 2013.

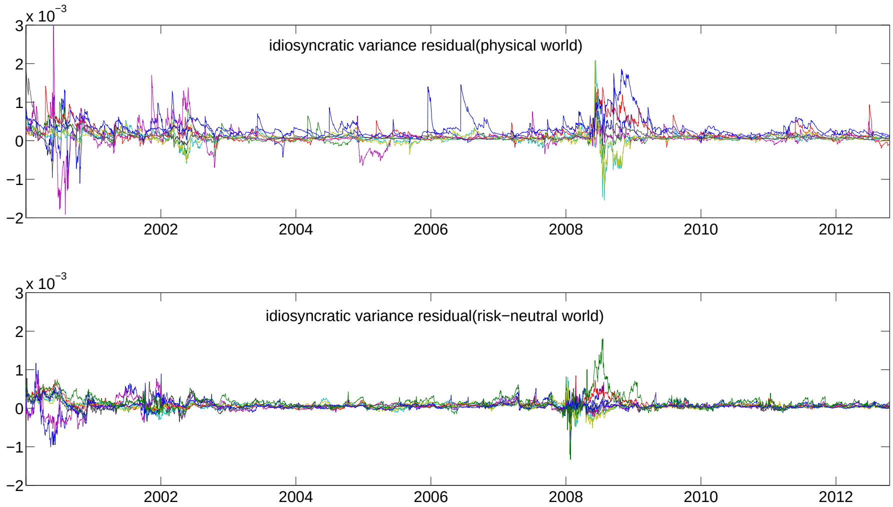

Figure 2: This figure shows variance residuals for 10 selected stocks in P and $\mathbb{Q}$ world. The variance betas are used as the factor loadings in both cases. Data window ranges from 3rd January 2000 to 31 August 2013.

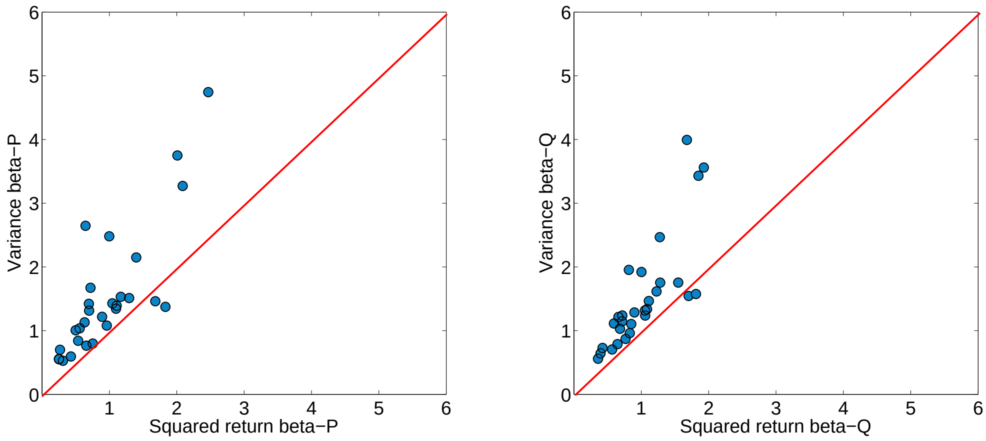

Figure 3: This figure shows the scatter plot of the variance betas versus the squared return betas in the P and the $\mathbb{Q}$ world. The $y = x$ line is drawn in the red line. In both graphs, 28 out of 30 stocks have higher variance betas than the squared return betas.

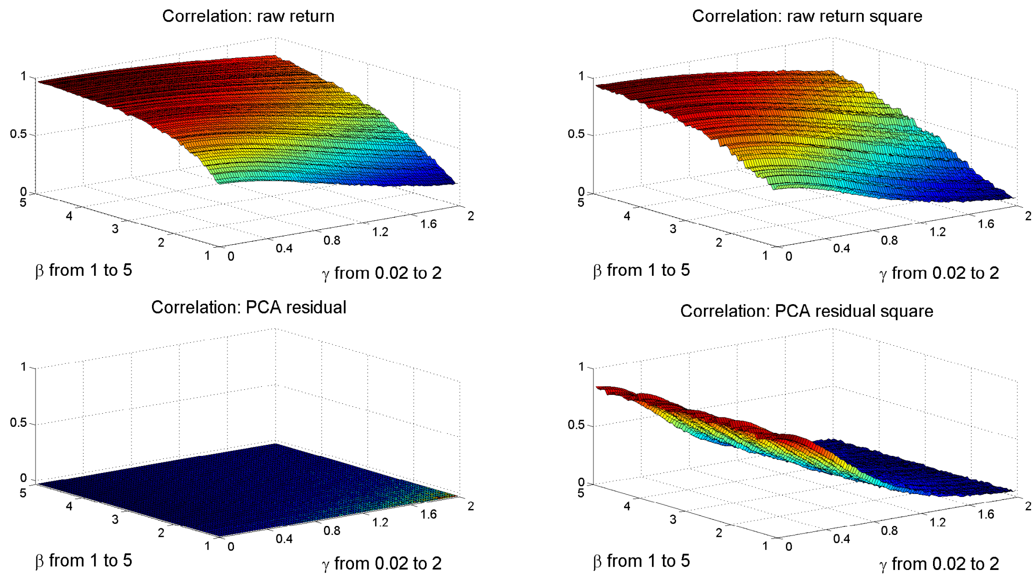

Figure 4: This figure shows the correlation of the returns, squared returns, the residuals from PCA, and the squared residuals. β ranges from 1 to 5; γ ranges from 0.02 to 2; Number of stocks is 30.

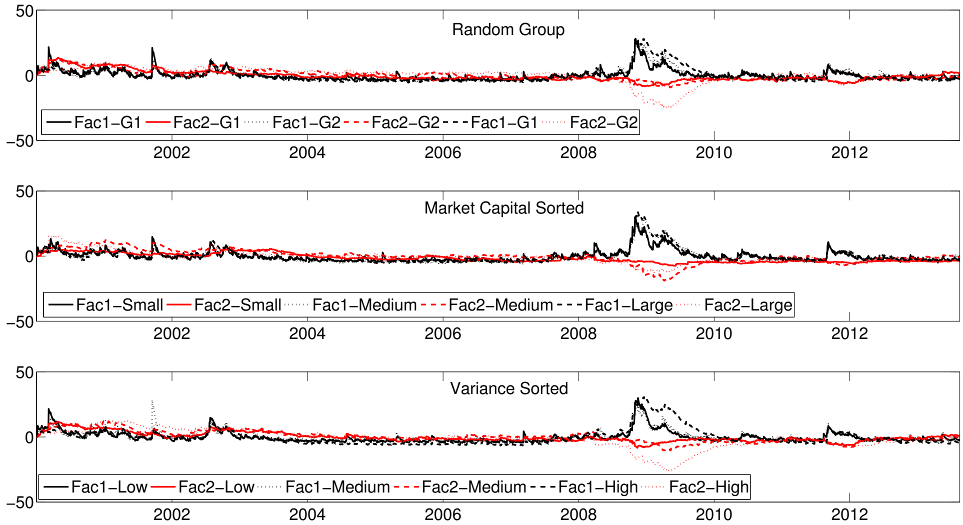

Figure 5: This figure shows the estimated factors from three groups of stocks sorted by random chosen, market capital sizes, and unconditional variances. Data window ranges from 3rd January 2000 to 31st December 2013. In each graph, Fac1 stands for the market variance factor, and Fac2 stands for the CIV factor. Each graph presents one sorting schemes; the numbers following the letter G in the legends denote the group numbers.

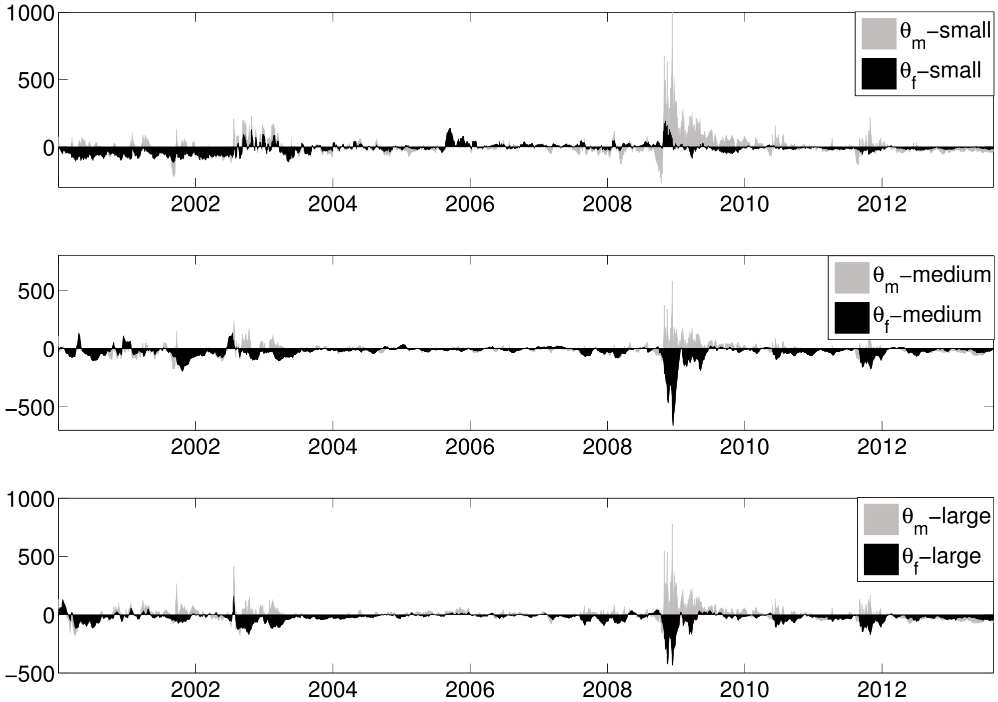

Figure 6: This figure shows the factor premium in the three groups of stocks sorted by market capital size. The black bar represents the factor premium of the CIV factor, the grey bar represents the factor premium of the market variance factor. Data window ranges from 3rd January 2000 to 31 August 2013.

Table 1: Summary of comovement results

Data	Model	Res.	Idio.Variance	Var.Res	Abs.Var.Res
P-world	Static	0.0089	0.5296	0.2704	0.3552
	Dynamic	0.0065	0.5208	0.1884	0.3760
Q-world	Static	–	0.5743	0.3515	0.4110
	Dynamic	–	0.6710	0.3159	0.4304

Note: This table presents pairwise average correlation for the variance residual when the market return factor being extracted by the return beta, and the variance residual after the market variance factor being extracted by the variance beta. The average correlation of the raw return series is 0.3957.

Table 2: Empirical estimates with sorted market capital size

Small	δ_{mi}	δ_{xi}	α_i	Medium	δ_{mi}	δ_{xi}	α_i	Large	δ_{mi}	δ_{xi}	α_i
TRV	0.5032	0.1559	3.6517	UTX	0.3929	0.2210	3.1850	PFE	0.2904	0.2712	3.1297
	(0.0743)	(0.0678)	(0.5666)		(0.0512)	(0.0699)	(0.4650)		(0.0387)	(0.0768)	(0.2892)
CAT	0.5877	-0.0612	4.2163	CSCO	0.8480	0.7375	7.3882	VZ	0.3984	0.4160	3.3304
	(0.0691)	(0.0634)	(0.4125)		(0.1098)	(0.1058)	(1.0965)		(0.0585)	(0.1321)	(0.3608)
DD	0.4401	0.0102	2.9931	HD	0.7511	0.5477	5.0315	CVX	0.2425	0.0095	2.4704
	(0.0528)	(0.0243)	(0.3200)		(0.1051)	(0.1022)	(0.7781)		(0.0278)	(0.0212)	(0.1524)
NKE	0.4257	0.1453	4.1059	DIS	0.5124	0.3180	4.0926	JPM	1.1825	0.1074	6.4865
	(0.0873)	(0.0715)	(0.6362)		(0.0707)	(0.0812)	(0.5404)		(0.1056)	(0.1318)	(0.6882)
GS	0.9879	-0.1265	5.3484	C	1.6405	-0.7097	8.9055	PG	0.2426	0.2710	2.1792
	(0.1162)	(0.0601)	(0.6502)		(0.2213)	(0.1807)	(1.5006)		(0.0387)	(0.0891)	(0.3225)
BA	0.5266	0.1268	4.2126	MRK	0.4501	0.2458	4.2012	WMT	0.3148	0.3284	2.7784
	(0.0956)	(0.0917)	(0.3955)		(0.0778)	(0.0542)	(0.5977)		(0.0395)	(0.0893)	(0.2807)
MCD	0.2155	0.4528	3.5757	IBM	0.3552	0.2544	2.8715	GE	0.6294	0.1269	3.8500
	(0.0672)	(0.1193)	(0.9883)		(0.0468)	(0.0426)	(0.3540)		(0.0553)	(0.0855)	(0.3688)
UNH	0.7693	-0.1245	4.7041	INTC	0.7870	0.8125	6.3991	JNJ	0.2286	0.2875	1.9338
	(0.1554)	(0.1370)	(0.6036)		(0.1275)	(0.1210)	(0.9737)		(0.0281)	(0.0642)	(0.2538)
AXP	1.0082	-0.0966	4.7816	KO	0.2891	0.2173	2.1536	MSFT	0.5195	0.4438	4.5803
	(0.1246)	(0.0813)	(0.5281)		(0.0438)	(0.0327)	(0.3095)		(0.0658)	(0.1412)	(0.4935)
MMM	0.2458	0.0554	2.2686	T	0.4821	0.4807	3.5121	XOM	0.2217	0.0613	2.3088
	(0.0468)	(0.0916)	(0.1565)		(0.0841)	(0.0712)	(0.5570)		(0.0242)	(0.0320)	(0.1310)
S&P500	0.2127		1.2652	S&P500	0.1905		1.3136	S&P500	0.2179		1.3348
	(0.0229)		(0.1233)		(0.0192)		(0.1392)		(0.0230)		(0.1138)
Average	0.5710	0.0538	3.9858	Average	0.6508	0.3125	4.7740	Average	0.4270	0.2323	3.3047

Note: This table presents the estimated factor loadings and the unconditional variances of all 30 stocks from three groups of 10 sorted by their market capital size. The standard errors are provided in brackets. Data window ranges from 3rd January 2000 to 31st December 2013.

Table 3: Empirical estimates with sorted unconditional variances

Low	δ_{mi}	δ_{xi}	α_i	Medium	δ_{mi}	δ_{xi}	α_i	High	δ_{mi}	δ_{xi}	α_i
JNJ	0.2023	0.1468	1.5484	IBM	0.4371	0.1811	2.7039	DIS	0.5017	0.3382	4.7673
	(0.0115)	(0.0192)	(0.0959)		(0.0216)	(0.0244)	(0.1170)		(0.0310)	(0.0404)	(0.2516)
KO	0.3353	0.2091	2.1530	T	0.4172	0.2901	2.8561	CAT	0.4490	0.0216	4.8120
	(0.0179)	(0.0243)	(0.1424)		(0.0228)	(0.0315)	(0.1587)		(0.0164)	(0.0220)	(0.0884)
PG	0.2407	0.2045	1.9624	MRK	0.4799	0.0908	3.8570	UNH	0.5410	-0.0366	5.3513
	(0.0179)	(0.0259)	(0.1272)		(0.0182)	(0.0204)	(0.1187)		(0.0256)	(0.0293)	(0.1020)
MMM	0.3009	-0.0020	2.3534	DD	0.4790	0.0184	2.9570	HD	0.5608	0.4079	5.1839
	(0.0205)	(0.0182)	(0.0957)		(0.0212)	(0.0216)	(0.1018)		(0.0422)	(0.0473)	(0.2917)
WMT	0.2902	0.2141	2.4595	UTX	0.4149	0.1207	2.7526	AXP	0.6370	-0.0172	4.8763
	(0.0175)	(0.0269)	(0.1404)		(0.0230)	(0.0217)	(0.1013)		(0.0216)	(0.0273)	(0.0862)
MCD	0.2708	0.3354	2.8127	TRV	0.5725	0.1195	3.3139	GS	0.6818	-0.0267	5.6208
	(0.0222)	(0.0401)	(0.1945)		(0.0198)	(0.0262)	(0.1220)		(0.0279)	(0.0327)	(0.1000)
XOM	0.3089	-0.0823	2.2911	NKE	0.4929	0.1422	3.8119	INTC	0.7275	0.6616	7.3658
	(0.0205)	(0.0226)	(0.1049)		(0.0191)	(0.0249)	(0.1254)		(0.0493)	(0.0678)	(0.4480)
CVX	0.3825	-0.1683	2.5304	BA	0.5340	0.0264	3.7379	CSCO	0.6952	0.6789	8.3995
	(0.0278)	(0.0278)	(0.1449)		(0.0241)	(0.0271)	(0.1192)		(0.0385)	(0.0606)	(0.4491)
VZ	0.3490	0.3311	2.9697	GE	0.6283	-0.0406	3.4095	JPM	0.9199	-0.0003	6.2365
	(0.0264)	(0.0410)	(0.2004)		(0.0274)	(0.0246)	(0.1378)		(0.0219)	(0.0333)	(0.3547)
PFE	0.2904	0.1139	2.8477	MSFT	0.4509	0.2428	3.5828	C	1.1636	-0.4581	7.4529
	(0.0194)	(0.0251)	(0.1143)		(0.0201)	(0.0316)	(0.1486)		(0.0421)	(0.0554)	(0.2278)
S&P500	0.3055		1.5334	S&P500	0.1964		1.2001	S&P500	0.1555		1.3094
	(0.0158)		(0.0876)		(0.0083)		(0.0424)		(0.0062)		(0.0250)
Average	0.2971	0.1302	2.3928	Average	0.4907	0.1191	3.2983	Average	0.6878	0.1569	6.0066

Note: This table presents the estimated factor loadings and the unconditional variances of all 30 stocks from three groups of 10 sorted by their unconditional variances. The standard errors are provided in brackets. Data window ranges from 3rd January 2000 to 31st December 2013.

Table 4: Empirical estimates with random group

RG1	δ_{mi}	δ_{xi}	α_i	RG2	δ_{mi}	δ_{xi}	α_i	RG3	δ_{mi}	δ_{xi}	α_i
MSFT	0.5351	0.3916	4.3977	BA	0.4999	0.1306	3.9656	AXP	0.8900	-0.0976	5.2816
	(0.0263)	(0.0545)	(0.3270)		(0.0336)	(0.0322)	(0.2012)		(0.0284)	(0.0495)	(0.1216)
KO	0.3853	0.2289	2.4244	PFE	0.2520	0.2093	2.8121	INTC	1.0996	0.8077	6.4530
	(0.0166)	(0.0252)	(0.1834)		(0.0139)	(0.0269)	(0.1549)		(0.0470)	(0.0744)	(0.3844)
DD	0.5652	-0.0373	3.2104	JNJ	0.1912	0.2008	1.6307	TRV	0.6524	0.2496	3.7945
	(0.0232)	(0.0212)	(0.1267)		(0.0117)	(0.0218)	(0.1283)		(0.0370)	(0.0360)	(0.1765)
XOM	0.3259	-0.0826	2.2365	MMM	0.2346	0.0613	2.1814	VZ	0.5338	0.3445	3.0010
	(0.0162)	(0.0190)	(0.0946)		(0.0124)	(0.0135)	(0.0957)		(0.0227)	(0.0328)	(0.1674)
GE	0.7760	-0.0282	3.8868	MRK	0.3943	0.2247	3.8839	T	0.6096	0.4263	3.2715
	(0.0318)	(0.0255)	(0.1678)		(0.0172)	(0.0236)	(0.2093)		(0.0135)	(0.0379)	(0.1443)
IBM	0.5036	0.1705	3.1279	DIS	0.5345	0.2809	4.0121	HD	0.7703	0.4266	4.5096
	(0.0207)	(0.0235)	(0.1726)		(0.0289)	(0.0362)	(0.2417)		(0.0361)	(0.0453)	(0.2346)
CVX	0.3397	-0.1349	2.3966	MCD	0.2600	0.3023	2.7090	C	1.2149	-0.6748	7.9961
	(0.0186)	(0.0246)	(0.1212)		(0.0168)	(0.0321)	(0.1948)		(0.0523)	(0.0793)	(0.3576)
UTX	0.5295	0.1841	3.3280	JPM	1.2439	-0.0227	6.1871	CSCO	1.0462	0.7175	7.4050
	(0.0236)	(0.0289)	(0.1809)		(0.0528)	(0.0253)	(0.4539)		(0.0421)	(0.0621)	(0.3378)
PG	0.2485	0.2517	2.1945	WMT	0.2361	0.2147	2.3415	GS	0.7556	-0.0949	5.5409
	(0.0141)	(0.0323)	(0.1959)		(0.0143)	(0.0245)	(0.1486)		(0.0367)	(0.0449)	(0.1600)
CAT	0.6545	-0.1896	4.4233	NKE	0.4231	0.1928	3.9287	UNH	0.7392	0.0597	5.3742
	(0.0346)	(0.0393)	(0.1971)		(0.0212)	(0.0281)	(0.1926)		(0.0471)	(0.0429)	(0.1935)
S&P500	0.2714		1.3962	S&P500	0.2241		1.2773	S&P500	0.1798		1.2713
	(0.0099)		(0.0558)		(0.0089)		(0.0761)		(0.0093)		(0.0370)
Average	0.4863	0.0754	3.1626	Average	0.4270	0.1795	3.3652	Average	0.8312	0.2165	5.2627

Note: This table presents the estimated factor loadings and the unconditional variances of all 30 stocks from three random groups of 10. The standard errors are provided in brackets. Data window ranges from 3rd January 2000 to 31st December 2013.

Table 5: Kalman Filter result

	$\hat{\mu}_m^{\mathbb{Q}}$	$\hat{\mu}_f^{\mathbb{Q}}$	$\hat{\psi}_m$	$\hat{\psi}_f$	$\bar{\lambda}$	$\bar{\lambda}_m$
Small (MC)	15.0196	-15.8270	0.9379	0.9700	8.9078	11.6834
	(6.5411)	(5.2203)	(0.0061)	(0.0045)	(6.7397)	(26.8355)
Medium (MC)	12.4420	-34.3582	0.9792	0.9875	0.6052	11.8610
	(13.8079)	(12.6665)	(0.0037)	(0.0028)	(8.9689)	(25.2311)
Large (MC)	32.7008	-26.4527	0.9506	0.9813	2.2460	9.8806
	(8.9267)	(8.5516)	(0.0056)	(0.0035)	(5.2552)	(21.6564)
Low (Vol)	8.5549	-0.2276	0.9003	0.9610	3.6648	6.7002
	(4.9729)	(3.1485)	(0.0079)	(0.0053)	(2.9919)	(24.7251)
Medium (Vol)	11.9963	25.1850	0.9472	0.9808	6.5610	12.4500
	(6.4540)	(17.0126)	(0.0057)	(0.0039)	(8.7470)	(28.4389)
High (Vol)	3.7225	-31.4894	0.9563	0.9693	2.7374	10.9289
	(8.3810)	(10.5933)	(0.0053)	(0.0045)	(12.2765)	(26.1647)
RG1	1.6828	-12.5246	0.9471	0.9836	2.0921	8.7586
	(7.1207)	(11.2607)	(0.0056)	(0.0036)	(4.6135)	(26.5027)
RG2	1.9891	4.1565	0.9585	0.9795	4.3852	11.3016
	(8.9683)	(8.2864)	(0.0054)	(0.0040)	(7.2325)	(24.8996)
RG3	14.4611	-14.2635	0.9689	0.9828	5.0069	11.9329
	(10.2031)	(14.9128)	(0.0044)	(0.0033)	(10.2227)	(25.5713)

Note: This table presents the estimated factor premia for both the market variance factor and the CIV factor. The persistent parameters are also provided together with the average premia for individual stock and index for each group. The standard errors are provided in brackets. Data window ranges from 3rd January 2000 to 31st August 2013.

Table 6: Beta comparison

	Beta-square	Var-Beta(RG)	Var-Beta(MC)	Var-Beta(Vol)	Ave. Diff
MSFT	1.0989	1.9715	2.3846	2.2956	1.1184
KO	0.2677	1.4197	1.5178	1.0975	1.0773
DD	1.0439	2.0823	2.0697	2.4391	1.1531
XOM	0.6953	1.2007	1.0177	1.0110	0.3812
GE	1.3990	2.8594	2.8891	3.1989	1.5835
IBM	0.7537	1.8554	1.8647	2.2255	1.2282
CVX	0.7201	1.2518	1.1129	1.2520	0.4855
UTX	0.9612	1.9510	2.0626	2.1124	1.0809
PG	0.2509	0.9156	1.1135	0.7878	0.6881
CAT	1.2935	2.4115	2.7634	2.8881	1.3942
BA	0.8923	2.2308	2.4760	2.7186	1.5829
PFE	0.5367	1.1244	1.3331	0.9505	0.5993
JNJ	0.2527	0.8533	1.0490	0.6623	0.6022
MMM	0.6569	1.0470	1.1559	0.9849	0.4057
MRK	0.4993	1.7596	2.3633	2.4434	1.6894
DIS	1.1694	2.3851	2.6902	3.2274	1.5982
MCD	0.3110	1.1601	1.0134	0.8863	0.7089
JPM	2.4677	5.5505	5.4278	5.9172	3.1641
WMT	0.4283	1.0536	1.4448	0.9501	0.7212
NKE	0.6994	1.8878	2.0019	2.5097	1.4337
AXP	2.0880	4.9506	4.7408	4.0975	2.5083
INTC	1.6820	6.1166	4.1319	4.6795	3.2940
TRV	0.9976	3.6288	2.3661	2.9148	1.9723
VZ	0.5597	2.9691	1.8288	1.1424	1.4204
T	0.6325	3.3906	2.5313	2.1240	2.0494
HD	1.1076	4.2846	3.9435	3.6075	2.8376
C	3.1238	6.7575	8.6132	7.4850	4.4948
CSCO	1.8322	5.8193	4.4523	4.4721	3.0824
GS	2.0099	4.2031	4.6452	4.3855	2.4013
UNH	0.6463	4.1118	3.6176	3.4803	3.0903

Note: This table presents the sample static beta square and the variance beta estimated in each sorting schemes. The average difference between the variance beta and the squared return beta is presented in the last column. Data window ranges from 3rd January 2000 to 31st August 2013.

Table 7: Portfolio performance

Strategy	Average Return	Std	Skewness	kurtosis	Average Monthly Return
Fac-strategy (small)	1.40%	0.1558	0.4127	65.3627	34.04%
Fac-strategy (medium)	2.45%	0.2993	18.3570	727.6777	66.42%
Fac-strategy (large)	0.92%	0.1269	-0.9337	103.2563	21.26%
Short Index Straddle	0.93%	0.1021	0.0961	45.1407	21.46%
S&P500 Index	0.01%	0.0132	-0.0052	11.0653	0.21%
$1/N$ on stock (small)	0.04%	0.0146	0.0700	12.0178	0.76%
$1/N$ on stock (medium)	0.03%	0.0212	21.3339	853.1123	0.72%
$1/N$ on stock (large)	0.01%	0.0125	0.1123	11.3577	0.26%

Note: This table presents the average daily excess returns for the factor-trading strategy in three groups sorted by market capital size. The standard deviation, skewness, kurtosis and the average monthly returns for each of the portfolio return time series of each strategy are also provided. Date window ranges from 3rd January 2000 to 31st August 2013.

Table 8: Portfolio weight

Strategy	Individual straddle	Individual stock	Index straddle	S&P500 Index	Risk-free
Fac-strategy (small)	100%	-40.66%	-160.71%	110.24%	91.13%
Fac-strategy (medium)	100%	-31.30%	-204.97%	161.36%	74.92%
Fac-strategy (large)	100%	-37.54%	-132.27%	90.99%	78.83%

Note: This table presents the average portfolio weight for the three groups of factor-trading strategies.